

Code No: 133BD

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year I Semester Examinations, April/May - 2023

MATHEMATICS - IV

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE, MIE, CEE, MSNT)

Time: 3 Hours

Max. Marks: 75

- Note:** i) Question paper consists of Part A, Part B.  
 ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.  
 iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

## PART – A

(25 Marks)

- 1.a) Define analytic function. [2]
- b) Find the limit of the function  $\lim_{z \rightarrow 2i} (3x + iy^2)$ . [3]
- c) Determine the poles of the integral  $\int_C \frac{4-3z}{z(2-z)} dz$ . [2]
- d) Explain about isolated singularity and removable singularity. [3]
- e) Write the cross ratio of four points  $z_1, z_2, z_3, z_4$ . [2]
- f) Find u and v values of the bilinear transformation  $w = \cosh z$ . [3]
- g) Define Fourier integral theorem. [2]
- h) Define the periodic function. [3]
- i) Write down the one-dimensional wave equation. [2]
- j) Explain the method of separation of variables. [3]

## PART – B

(50 Marks)

- 2.a) Check the continuity of  $f(z) = \begin{cases} \frac{z}{|z|} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$ . [5+5]
- b) Discuss the analyticity of the function  $f(z) = z\bar{z}$ . [5+5]

OR

- 3.a) Show that  $u = \frac{1}{2} \log(x^2 + y^2)$  is harmonic and find its harmonic conjugate function.
- b) Using definition of derivative, find  $f'(z)$ , if it exists for the following function  $f(z) = \frac{1+z}{(1-z)}, z \neq 1$ . [5+5]

- 4.a) Find the residue of  $f(z) = \frac{z^3}{z^2-1}$  at  $z = \infty$ .
- b) Evaluate  $\oint_C \frac{e^{2z}}{(z+i)^4} dz$  where C is the circle  $|z| = 3$ . [5+5]

OR

- 5.a) Evaluate the complex integral  $\oint_C \tan z dz$  where C is the circle  $|z| = 2$ .
- b) Find the Taylor's series expansion for the function  $f(z) = \left(\frac{z}{z+1}\right)^2$  at  $z_0 = 1$  about the point  $z_0$ . [5+5]

- 6.a) Using contour integration method prove the integral  $\int_0^{2\pi} \frac{x^{a-1}}{1+x} dx = \frac{\pi}{\sin\pi a}$ .
- b) Find the bilinear transformation such that the point  $z=i$  is the only fixed point and the point '1' is mapped as  $\infty$ . [5+5]

**OR**

- 7.a) Discuss fully the transformation  $w = c \cosh z$ , where  $c$  is a real number. What physical problem can we study with the help of this transformation?
- b) Evaluate  $\int_0^{\infty} \frac{\cos 3\theta}{5+4\cos\theta} d\theta$ . [5+5]

- 8.a) Evaluate the Fourier series expansion of  $f(x) = e^{-ax}$  in  $(0, 2\pi)$ .
- b) Find Fourier transform of  $f(x) = xe^{-x}$ , in  $0 \leq x \leq \infty$ . [5+5]

**OR**

- 9.a) Find Fourier sine and cosine transform of  $f(x) = 2x$  in  $0 < x < 4$ .
- b) Find Fourier series expansion of  $f(x) = \frac{(\pi-x)}{2}$  in  $0 < x < 2$ . [5+5]

- 10.a) Solve the equation by method of separation of variables  $4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$  and  $u(0, y) = e^{-5y}$ .
- b) Explain the possible solutions of one-dimensional heat equation. [5+5]

**OR**

11. A tightly stretched string with fixed end points  $x=0$  and  $x=l$  is initially in a position given  $y(x, 0) = y_0 \sin^3 \frac{\pi x}{l}$ . If it is released from rest position, find the displacement  $y$  at any time and any distance from the end  $x=0$ . [10]

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